

Sec 1.3 slope fields and solution curves.

An ODE $\frac{dy}{dx} = f(x, y)$ specifies a slope (L to R) at each point (a, b) . ("slope field")

Solution curves are the curves traced by the graphs of solutions of the ODE

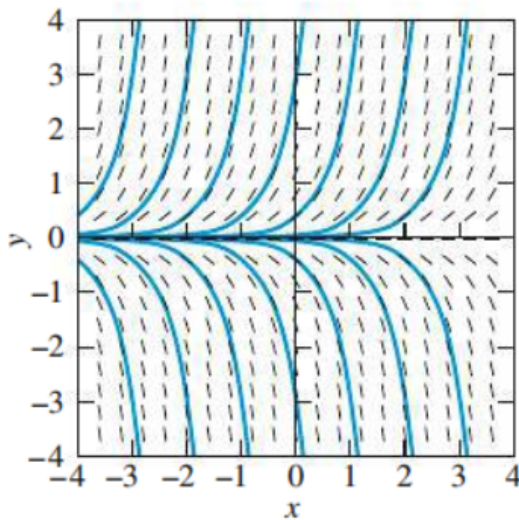


FIGURE 1.3.2(a) Slope field and solution curves for $y' = 2y$.

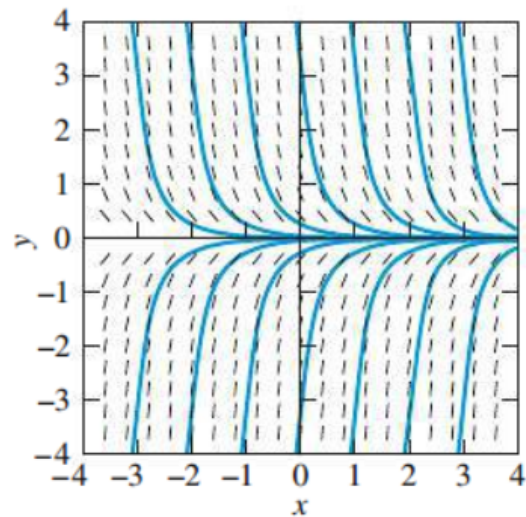
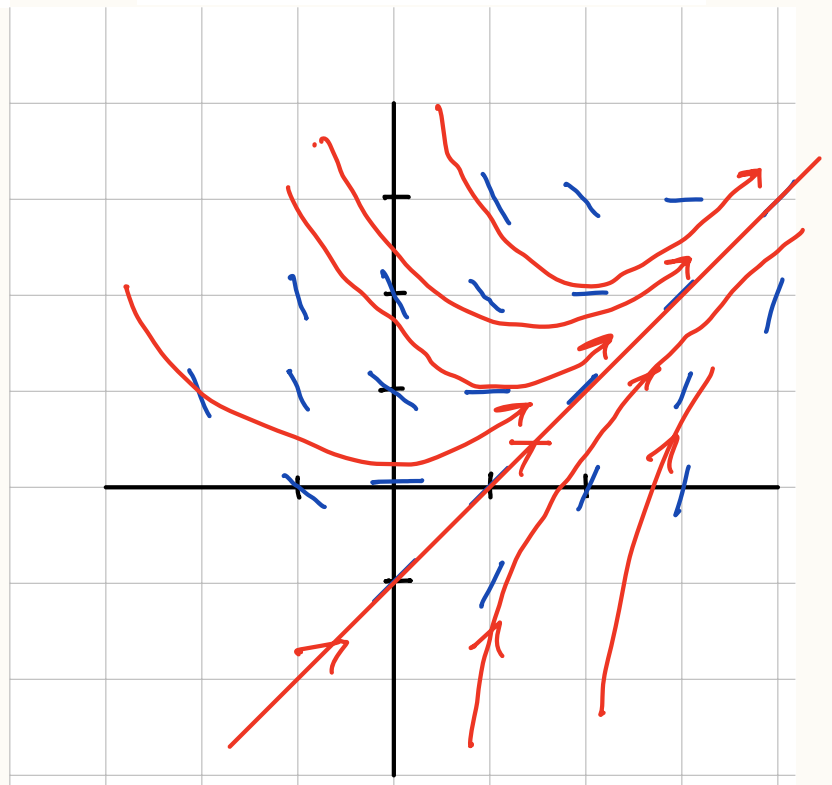


FIGURE 1.3.2(d) Slope field and solution curves for $y' = -3y$.

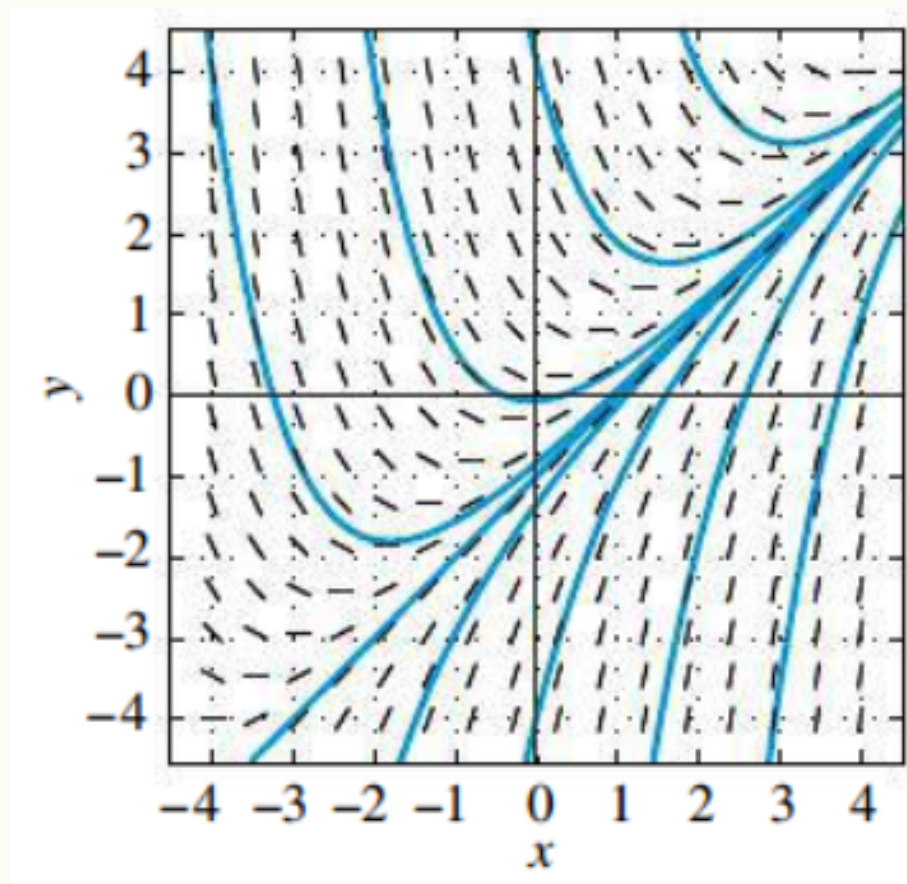
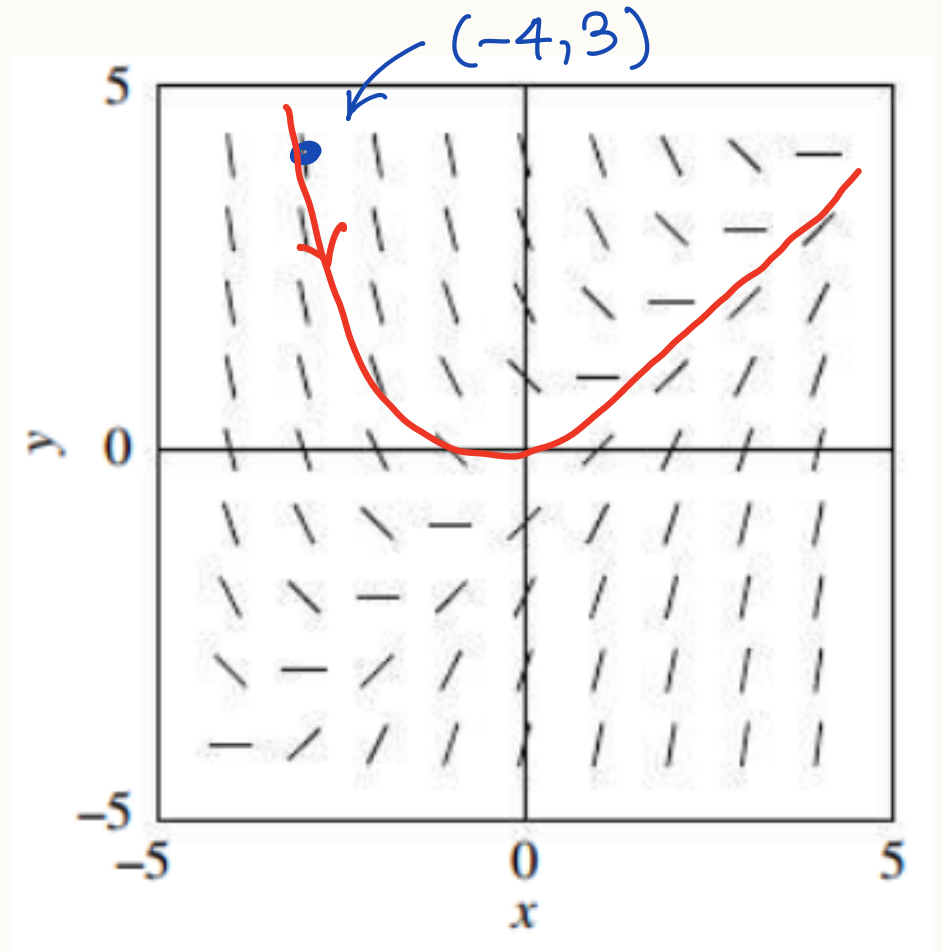
Ex: Sketch a slope field for $y' = x - y$ and some solution curves:

★ A solution $y(x)$ having initial value $y(a) = b$ means sol. curve must pass through (a, b)



Solution of $y' = x - y$ having $y(-4) = 3$

(graph of part.
sol. for IVP
 $\begin{cases} y' = x - y \\ y(-4) = 3 \end{cases}$)

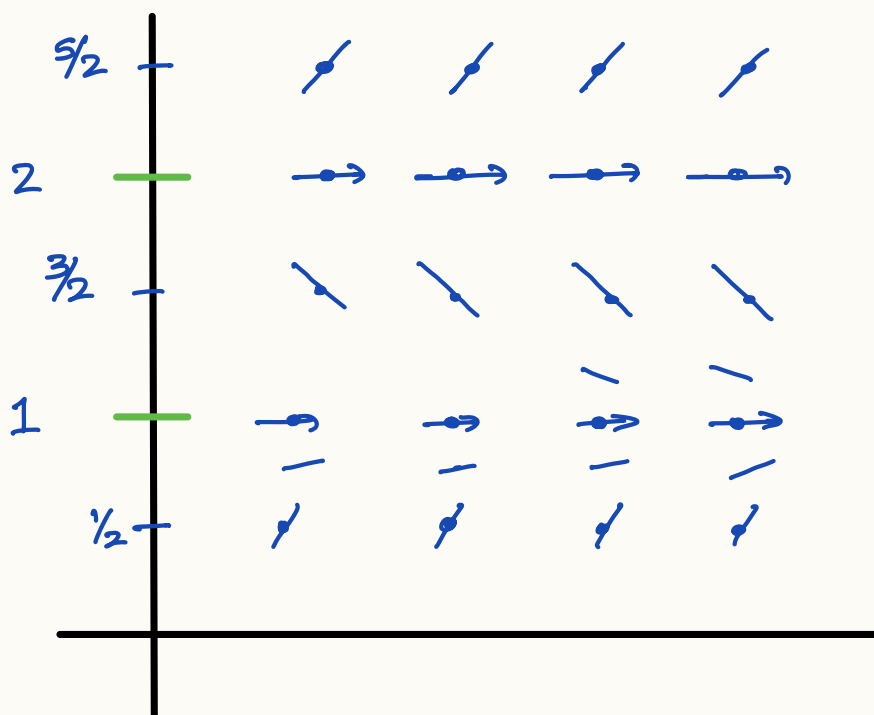


Ex what might a slope field for

$$y' = (y-1)(y-2)$$

look like in the region $x \geq 0$?

How about some solution curves?



Recall ideas from calculus 1:

$$y' = (y-1)(y-2) \Rightarrow y' = 0 \text{ when } y = 1 \text{ or } 2.$$

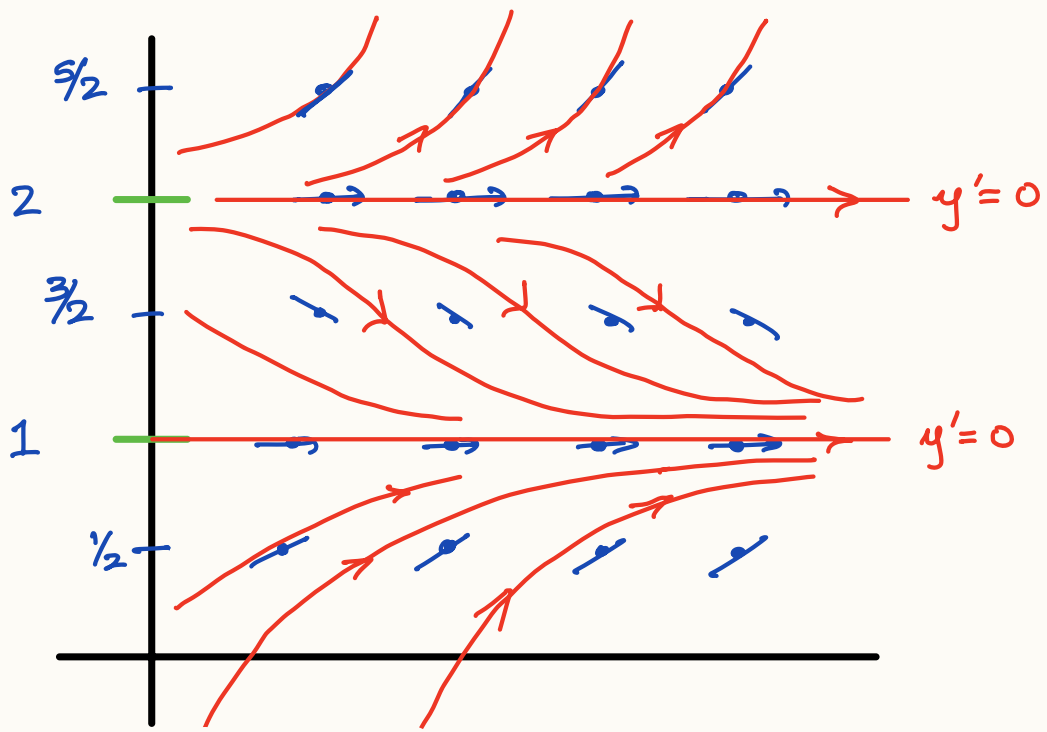
• Also, y' doesn't depend on x .

• If $y < 1$ (and < 2), $y' > 0$

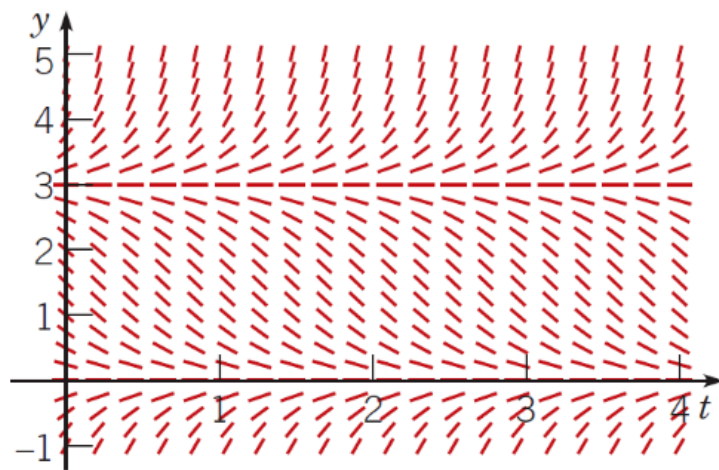
then $(y-1)(y-2) = (-)(-) = +$

• If $1 < y < 2$, then $y' = (+)(-) = -$

• If $y > 2$, then $y' = (+)(+) = +$



Among the following choices, which ODE has a slope field similar to the one to the right?



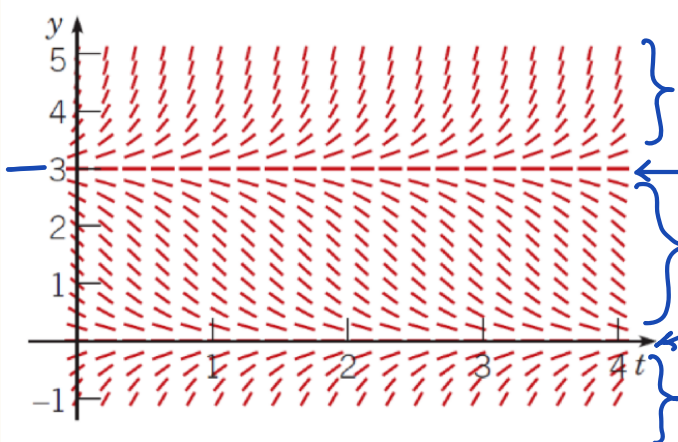
A. $y' = y(3 - y)$

D. $y' = y(y - 3)$

B. $y' = y - 3$

E. $y' = y + 3$

C. $y' = y(3 + y)$



increasing ($y' > 0$) when $y > 3$

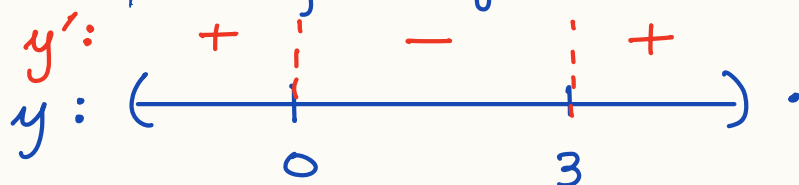
flat ($y' = 0$) when $y = 3$

decreasing when $0 < y < 3$

$y' = 0$ when $y = 0$

$y' > 0$ when $y < 0$

Thus our description of the function is



Only choices A and D have $y' = 0$ when $y = 0$

and $y = 3$. Choice A: $y' = y(3 - y)$ $\left\{ \begin{array}{l} \text{if } y < 0, y' = (-)(+) = - \quad \times \\ 0 < y < 3, y' = (+)(+) = - \quad \times \\ y > 3, y' = (+)(-) = + \end{array} \right.$

Choice D $y' = y(y - 3)$ $\rightarrow \left\{ \begin{array}{l} y < 0 \Rightarrow y' = (-)(-) = + \quad \checkmark \\ 0 < y < 3 \Rightarrow y' = (+)(-) = - \quad \checkmark \\ y > 3 \Rightarrow y' = (+)(+) = + \quad \checkmark \end{array} \right.$

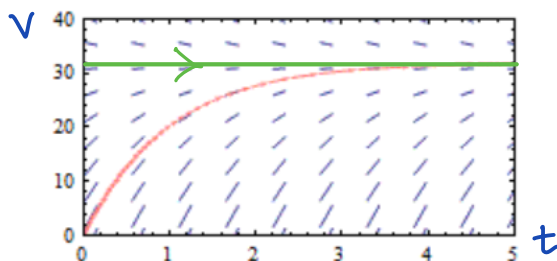
You bail out of a helicopter and pull the ripcord of your parachute. Now the air resistance proportionality constant is $k = 1.6$, so your downward velocity satisfies the initial value problem below, where v is measured in ft/s and t in seconds. In order to investigate your chances of survival, construct a slope field for this differential equation and sketch the appropriate solution curve. What will your limiting velocity be? Will a strategically located haystack do any good? How long will it take you to reach 95% of your limiting velocity.

$$\frac{dv}{dt} = 32 - 1.6v, v(0) = 0$$

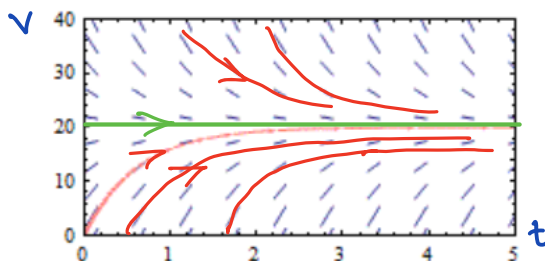
$\uparrow$ air resistance $\propto (-\text{vel.})$
 $g \downarrow = 32 \text{ ft/s}^2$

Construct a slope field for this differential equation and sketch the appropriate solution curve. Choose the correct graph below.

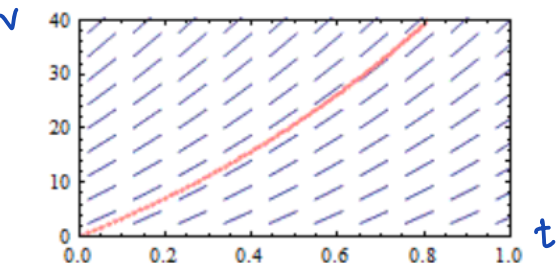
☒ A.



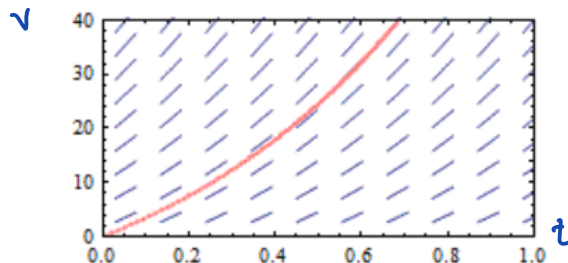
☐ B.



☐ C.



☐ D.



$$a(t) = g - kv \quad (\text{"down" is "+"})$$

$$v'(t) = 32 - 1.6v = 32 - \frac{16}{10}v = 16 \left(2 - \frac{1}{10}v \right)$$

$$\frac{dv}{dt} = \frac{8}{5}(20 - v) \quad \left(\text{careful that this isn't } \frac{dv}{dt} = f(t) \right) = \frac{16}{10}(20 - v)$$

$$\left\{ \begin{array}{ll} v < 20 \Rightarrow v' > 0 & (\text{speed inc.}) \\ v = 20 \Rightarrow v' = 0 & (\text{no change}) \\ v > 20 \Rightarrow v' < 0 \end{array} \right.$$

Thus, $v(t)$ tends toward $v = 20$ (terminal velocity)

This is $20 \text{ ft/sec} \approx 13.6 \text{ mph} \dots$ you'll live